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DIMENSIONLESS UNITS AND NUMBERS

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The theory of measurement scales is used to show that there is no foundation for attempting to extend the SI system to measurements of quantities and properties which are described by ordering and naming scales or by absolute scales. It is proposed that the units of planar and solid angles should be considered to be outside the system. Dimensionless quantities are conditionally classified. An analysis is made of specified order scales in which the concept of a "unit of measurement" is not applicable and for which it makes no sense to attribute dimensionality to the numbers and scale points used in them.

A number of publications have appeared in recent times which contain discussions of questions associated with dimensionless (relative) units which are widely used in metrological practice. Attempts are made to allot "zero dimensionalities" to them. Particular attention is devoted to the units for measuring angular quantities. An analysis of publications such as [1] convincingly shows that their authors are unfamiliar with the theory of measurement scales [2, 3] and with a number of articles published in periodical journals dealing with these questions [4–7]. Since an incorrect understanding of the problems discussed can have an adverse effect on practical metrology and can form erroneous concepts in the minds of students and junior metrologists we shall attempt to introduce the necessary clarity.

It can be assumed that the primary factor in the appearance of real and invented difficulties in the use of dimensionless quantities and their units is the superficial and somewhat incorrect idea of the role and place of the SI system of international units in contemporary metrology, science, and technology [8]. The proposition familiar from school that the metric system, whose present-day embodiment is the SI system, has been "created for all time and for all people" is frequently understood as implying its uniqueness and universal sufficiency, and this does not correspond to reality. We shall not be concerned with the fact that some countries still use the foot-pound-second system and we shall formulate the limitations inherent in the SI system independently of the time and place of its use.

The principal limitation follows from the actual name "system of units." In the language of the theory of scales this denotes that the SI system is not extended (and cannot be extended) to properties and quantities describing nonmetric naming and ordering scales which do not possess units of measurement [4, 8, 9]. Moreover, the requirement of forming dimensionalities of derived units out of symbols allotted to the basic SI units fails to leave room for dimensionless quantities and their units in this system. And the only principle recognized by the SI system of forming multiples and fractions of units which does not include many widely used traditional units (such as the minute, the hour, the day, etc.) predetermined the existence of quite a large set of units outside the system which are used on a par with SI units.

It is necessary to precede a further exposition with a detailed consideration of the concept and term "dimensionality." There is a quite clear and unambiguous definition of "dimensionality" that it is an expression in the form of a monomial composed of the products of the symbols of basic quantities in various powers and reflecting the relationship of a given quantity to quantities adopted in this system as its basic units and to a coefficient of proportionality equal to unity [10]. Despite this definition, which fails to contain any allusions to the fact that it is assumed that dimensionality reflects a deep physical essence of some particular quantity, such investigations continue [1]. Attempts are also being made to attribute dimensionality to fundamentally dimensionless quantities. Our doubts concerning the competence of authors of such investigations are not unsubstan-

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tiated. It is sufficient to say that they fail to distinguish and confuse the dimensionality of units of measurement with their expressions in terms of the basic units of the system and even with their names (for example, see [1]). The authors are evidently not aware of the statements of such scientists as Max Planck, P. Bridgman, and others. Max Planck wrote the following: "... it is clear that the dimensionality of any physical quantity is not a property associated with its essence but simply represents some arbitrariness determined by the choice of the system of measurements" [7, 11, 12]. This point of view is confirmed by the dependence of dimensionalities on the chosen system of units, by the coincidence of dimensionalities possessing different physical natures, by dimensionalities of a number of quantities which are difficult to interpret physically, and by the fact that quantities which are dimensional in one system may be dimensionless in another (and *vice versa*). We finish this digression on the dimensionalities of measured quantities with the following quotations from [13]: "... in a case when only one set of basic units of measurement is taken, the dimensionality formula ... can be expressed by only one method. However, when the basic units are changed, the form of the formula can also change. It should be noted that whereas two dimensionality formulas cannot correspond to one physical quantity, one dimensionality formula can correspond to two different physical quantities" (page 29). Further: "No such concept as the absolute dimensionality of a physical quantity exists. ... Dimensionalities are relative by definition. We hold to the following point of view. A dimensionality formula of a physical quantity is based on a definition of this quantity which in itself depends on the method of measuring the quantity utilizing basic units of measurement whose choice (within certain limits) is arbitrary" (p. 133). It can be seen from this that dimensionality symbols are specific logical operators which are functionally defined only as part of the corresponding units of measurement. It must not be forgotten that it makes no sense to use these operators outside systems of units. One must also not speak of "the magnitude of dimensionalities" [1], since dimensionality symbols are not ordinary quantities and the abstract algebra of operations with them differs from ordinary algebra.

How does one then understand dimensionless quantities and their units? The theory of measurement scales makes it possible to give a clear answer to this question. Dimensionless quantities expressed by abstract numbers can be divided into two classes: absolutely dimensionless and conditionally dimensionless. Absolutely dimensionless quantities are quantities described by absolute scales (see Table 1). Absolute scales possess natural units of measurement (which are independent of any system of units). In all else they are similar to metric ratio scales [5, 9]. Absolute scales and their units can be realized without standards but such standards may exist in technically and economically well-founded cases. Any units of absolute scales are dimensionless since they are defined without relation to any system of units, although the units of absolute scales are associated with any systems of units as units outside the system. There is no need to include them in any system as system units. They are closely harmonized with SI units and units of other systems. They are essentially units outside or even above the system [14].

Conditionally dimensionless quantities (see Table 2) are dimensional quantities, transformed by dividing dimensional quantities by some fixed (reference) values of the same quantities. The logarithms of such ratios form logarithmic scales with a fixed zero [15]. Thus, it is possible to express the values of dimensional quantities in dimensionless units. The addition or subtraction of quantities expressed in such logarithmic units reduces to a determination of the logarithm of the sum or difference of the quantities whose logarithms are known.

A search for the dimensionalities of units of absolute scales is useless and futile. Enlisting the equations of fundamental physical laws to such attempts does not and cannot help [1]. For example, ascribing the dimensionality $(L^2MT^{-2})^0$ to efficiency and introducing the corresponding units (joules to the power zero) does not add any clarity and is simply inconvenient. Is it the case for example that an efficiency of 0.5 is half a joule to the power zero? Why? Here, without belittling the role of dimensionalities, it is necessary to remember that in practice what interests us is not the dimensionalities as such but the expressions linking the units of measurement with the basic units of the system and with each other. Their structure is similar but they are not identical. Sometimes they are not distinguished and this leads to the typical errors which also characterize [1]. It is not by chance that there is no "dimensionality" column in tables of an international document [16], only expressions for the relationship between the different units of measurement being given, since it is these which are widely used in practice. The fears of the author of [1] that confusion can arise from the use of dimensionless units are far-fetched and unfounded. This problem has never arisen and moreover could not arise. The specific application of dimensionless units is simply manifested by the name of the quantity (efficiency, gain, Q-factor, etc.) or by its designation. An arithmetic unit is never written after numerical values of a dimensionless quantity; there is no point in writing one.

Let us now consider yet another question which causes particular worry [1]. This concerns the planar and solid angles. This question is indeed of a specific nature, but is not as complex and confused as this author considers. In 1960, the category of "supplementary units" was introduced into the SI system and the radian and steradian were included in it. The fact that the

TABLE 1. Units of Absolute Scales

Quantity	Natural measurement unit	Multiples and fractions			
		Name	Notation		Ratio to natural unit
			Russian	international	
Relative quantities (ratio of similarly named quantities)*	Arithmetic (dimensionless) unit, "1" (not written after a numerical value of a quantity)	Percent Per thousand Per hundred thousand Millionth	% ‰ ‱ mil^{-1}	% ‰ ‱ ppm	10^{-2} of the dimensionless unit (d.u.) 10^{-3} d.u. 10^{-5} d.u. 10^{-6} d.u.
Planar angle, geographic latitude and longitude, phase of harmonic oscillations	Total angle (one revolution), period, cycle	Radian Angular degree Angular minute Angular second Metric degree Right angle Rumb Mil	rad ...° ...′ ...″ grad (gon) ...∟ rumb T	rad ...° ...′ ...″ gon or $\ldots^{g}$ ...∟ R 	$1/2\pi$ of total angle (full revolution) 1/360 of full revolution 1/21600 of full revolution 1/1296000 of full revolution 1/400 of full revolution 1/4 of full revolution 1/32 or 1/16 of full revolution respectively in navigation and meteorology $1/(2\pi \cdot 10^3)$ of full revolution
Solid angle	Total solid angle (sphere, assembly of all elementary solid angles in all directions in space around a point)	Steradian Square degree	sr □°	sr □°	$1/4\pi$ of total solid angle $(\pi/180)^2$ of total solid angle
Logarithmic quantities (logarithms of ratios of arbitrary values of quantities of the same kind)	Arithmetic unit of logarithm of relative quantity	Bel Decibel Neper	B dB Np	B dB Np	$\lg 10 = 1$ 0.1 B $\ln e = 1$
Frequency intervals	Arithmetic unit	Decade Octave	Dek Oktava		Ratio of limiting frequencies: 10 for a decade, 2 for an octave
Accounting (piece) quantities	Arithmetic unit	Einstein Item Pair Ten Dozen Gross Thousand Million units of goods**	É sht para desyatok dyuzhina gross tys mln	E	$6.0221367 \cdot 10^{23}$ d.u. 1 d.u. 2 d.u. 10 d.u. 12 d.u. 144 d.u. 10^3 d.u. 10^6 d.u.
Information content	Binary arithmetic unit	Bit Byte Kilobyte Megabyte Gigabyte Nat	bit bait kbait Mbait Gbait nat	bit B KB MB GB –	$\log_2 2 = 1$ 8 bits 1024 B 1024 kB 1024 MB 0.693 bits
Refractive index	Arithmetic unit	–	–	–	–
Similarity criteria (Reynolds number, Froude number, Mach number, Knudsen number, Euler number, etc.)	Arithmetic unit	–	–	–	–

* Examples of relative quantities: efficiency; Q factor; coefficients of transmission, reflection, absorption, attenuation, gain; albedo; modulation depth, etc.
** Accounting units associated with the name of a commodity: cans, bobbins, manufactured components, coils, packs, sets, containers, canisters, consignments, rolls, packages, tanks, copies, boxes, etc.

SI is the only system in which such a category is present and that there is no definition of this term has been emphasized many times in the literature. This situation has given grounds for discussions [10]. These resulted in the exclusion from the SI system of a separate category of "supplementary units" and their accommodation in the international standard ST ISO 1000-1992 of the International Standards Organization with a reference to an amendment by the 1980 International Conference on Weights and Measures to the table "Specially named derived units including supplementary SI units" [6, 17]. In 1995, a resolution of the Twentieth State Conference on Weights and Measures proposed that the radian and steradian be "interpreted" as dimensionless derived SI units [16]. This resolution is formally acceptable, but it would have been preferable directly to name the radian and steradian as off-system dimensionless units utilized in the SI system.

In this case, the theory of measurement scales gives a simple and clear explanation. The radian and steradian, as units of absolute scales, are not related to the basic SI units. They are typical units outside (or above) the system. It is now difficult to reestablish the logic of the arguments which led the International Conference on Weights and Measures to decide to include the radian and steradian with SI units, but the tendency to an all-embracing nature of the SI system is clear.

In addition to the search for the dimensionalities of angular units by the author of [1], there is concern over the application in practice of degree units along with the radian (we mainly discúss units for measuring planar angles) and the possibility of confusion arising (as in general with the units of absolute scales which we discussed above). Strangely enough, the author mentioned does not list all such units. Beside the radian and the angular degree (with its fractional units) the grad and compass points are in use (see Table 1). In military affairs (for example in artillery) mils are used. It is easy to remark that all these units originate from one natural unit, the full angle of revolution, the angle through which a material body rotates for all of its points to occupy their previous positions. In mathematical operations and their applications [18], in accordance with the definitions and properties of trigonometric functions, it is necessary to use only one radian unit, namely one period of revolution (a cycle) equal to 2π radians. In practical work it is preferable to use degree units. The values of planar angles are reproduced in degrees in the State Russian Standard GÉT 22-80. The reasons for this are generally known and there is no need to repeat them [6]. We just emphasize yet again than nobody has yet confused radians and angular degrees and that the conversion of the values of angles from one set of units to the other does not give rise to additional errors since the value of π is known with a clearly excess number of significant figures.

In order to supplement what has been said concerning the units of measurement of planar and solid angles, let us elucidate the other units in Table 1.

When forming relative units, one takes the ratio of the arbitrary values of similarly named quantities, and so the units of such quantities cannot in principle be associated with dimensional units. Widely used fractions of dimensionless units have the special names and notations indicated in Table 1. It is far rarer to use special notations of multiple dimensionless units to express the values of relative quantities. Absolute scales are used to describe a fairly large class of relative units such as coefficients of transmission, reflection, division, gain, and multiplication, Q factor, albedo, efficiency, etc.

The units of logarithmic quantities (the logarithms of relative quantities) also possess the indicated special names and notations. Examples of such logarithmic quantities are attenuation and optical density.

The distinctive units of frequency intervals, the decade and the octave, are given separately in Table 1. A whole series of dimensionless units for measuring frequency intervals are used in acoustics and music, in addition to these interval units [19]. These include the half-tone, full-tone, minor and major third, fourth, diminished and pure fifth, minor and major sixth, minor and major seventh, the Savart, the millioctave, the cent, the half-octave, third octave, and sixth octave. Frequency intervals distinguished by the ratio of the limiting frequencies are relative quantities. It should be noted that the linking of any frequency interval to a specific frequency value gives evidence of a quantity of another type, a conditionally dimensionless quantity (see Table 2).

The units of accounting quantities (the item, pair, ten, etc.) are widely used in accounting and trading operations encompassed by the spheres of State metrological monitoring and inspection. Agreements are known in the Council for Mutual Economic Aid and the Commonwealth of Independent States (CIS) concerning the accounting units whose use is permitted. A unit of measurement for the number of photons of optical radiation, the Einstein, was used in photochemistry. This is numerically equal to the Avogadro constant (number). Incidentally, the amount of a substance expressed in moles is also essentially an accounting quantity and therefore its unit of measurement could by agreement have been dimensionless (in the SI system the mole is one of the basic units with its own dimensionality, although there are no standards for the mole, nor will there be). Integer counting units are widely used in the SI system in order to form derived units of such quantities as frequency (Hz = 1 c/s), nuclide activity (Bq = $1\ s^{-1}$), the flux ($1\ s^{-1}$) and fluence ($1\ m^{-2}$) of ionizing particles, aerosol concentrations ($1\ m^3$), etc.

TABLE 2. Units of Conditionally Dimensionless Quantities

Quantity	Name	Measurement unit		
		Notation		Definition, reference quantity and its value
		Russian	international	
Level (energy quantities)	Logarithmic unit	dB (relative to P_0)	dB (re. P_0)	$10\lg(P/P_0)$ where P is the measured quantity and P_0 is the reference value, for example, 1 W for an electric power level or 10^{-12} W/m^2 for a sound intensity level
Level (force quantities)	Logarithmic unit	dB (relative to F_0)	dB (re. F_0)	$20\lg(F/F_0)$ where F is the measured quantity and F_0 is the reference value, for example, 1 mV for an electric voltage level or $2\cdot10^{-5}$ Pa for a sound pressure level
pH value	Unit of logarithm of dimensionless quantities	1	1	pH = $-\lg(m_H\gamma_H/m°)$ where m_H is the molarity of hydrogen ions, moles/kg; γ_H is the molar activity coefficient, dimensionless; $m°$ is the molarity of hydrogen ions in the standard state, equal to 1 mole/kg, the reference value
Relative permittivity ε	Arithmetic unit	1	1	$\varepsilon = \varepsilon_a/\varepsilon_0$ where ε_a is the absolute permittivity, F/m; ε_0 is the absolute permittivity of free space, equal to $8.854\cdot10^{-12}$ F/m, the reference value.
Sound pitch in music	Octave	Oktava	–	Ratio of octave limiting frequencies is 2. Reference value of sound pitch scale is the frequency 440 Hz, the note A of the first octave.
Note. The symbol 1 is usually omitted in association with a numerical value.				

It is possible to qualify a distinctive group of units of information content only as dimensionless units above the system. However, these units are naturally used in conjunction with SI units in order to form derived units such as the surface density of recording information (MB/cm^2).

Refractive index is also a relative quantity. It is the ratio of the velocity of light in a vacuum to that in the medium. A particular feature of this ratio quantity is that the numerator always contains a limiting velocity, and so the dimensionless value of a refractive index is always greater than unity. The latter remark does not apply to a relative refractive index (the ratio of light velocities in adjoining media). A new edition of a document of the International Organization of Legislative Metrology on SI [16] the refractive index and its unit (the number 1) are given in a table of examples of derived units expressed in terms of basic units (with a remark that the symbol "1" is usually omitted in combination with the numerical value). According to the theory of measurement scales, there is no justification for placing the refractive index in this table of the document [16].

The numerical values of dimensionless similarity criteria remain unchanged on passing from one system of units to another within the limits of a stipulated class of phenomena. This limitation must be kept in mind when using the theory of similarity and dimensionality.

In the State system for providing traceability of units of measurement there are more than 20 State standards and devices of higher accuracy which originated in Russia and which reproduce absolute scales (dimensionless units) of measurement. These standards include measurements of the following quantities: planar angle, volumetric moisture content, relative humidity of gases, grain moisture content, mass fraction of moisture in liquids, moisture content of photographic materials, humidity of disperse media, fraction of components in gases, mass fraction of matter, electrical Q factor, coefficient and angle of scale conversion, refractive index and its relative distribution, spectral transmission and reflection coefficients of optical radiation, phase shift angle of electric voltages, attenuation and phase shift of electromagnetic oscillations, attenuation of an optical signal, complex reflection coefficient of electromagnetic waves.

A typical representation of scales of conditionally dimensionless quantities (see Table 2) is given by logarithmic scales with a fixed zero determined by the initial (reference) value of the quantity [15]. The result of measurements in such scales are usually expressed in bels (B), decibels (dB), or nepers (Np) and are referred to as the level of the measured quantity. A measured value in these logarithmic units shows the extent to which a quantity has increased (by what factor its level has risen) rel-

TABLE 3. Order Scale Numbers and Points

Measured object	Measured quantity	
	Name	Notation
Metals and alloys	Hardness number on the scales: Brinell Rockwell Vickers Shore Yield point hardness, GOST (State All-Union Standard) 22762-77	 HB HRC HV HSD $H_{0.2}$
Minerals	Hardness numbers (points) on the Mohs scale	–
Rubbers	Hardness numbers on international scale	IRHD
Metals	Scratch microhardness, GOST 21318-75	$H_{\square p}$ $H_{\nabla p}$
Plastics	Hardness on Rockwell scale, GOST 24622-91 Hardness, GOST 4670-91	H_R HK
Paint coatings	Hardness, GOST 5233-89	–
Chipboards	Hardness, GOST 11843-76	H
Coal and anthracite	Microhardness, GOST 21206-75	H
Grinding tool	Degree of hardness on discrete scale, GOST 19202-80	–
Photographic materials	Light sensitivity, GOST 9160-91 Monochromatic sensitivity, GOST 2818-91	S S_λ
Motor fuel	Octane number GOST 8226-82 and GOST 511-82	–
Diesel fuel	Cetane number, GOST 3122-67 Cetane index, GOST 27768-88	– –
Petroleum products	Acid number, GOST 5985-79 Iodine value, GOST 2070-82	– X
Petroleum products and lubricating oils	Neutralization number, GOST 29255-91	–
Residual fuel	Bromine number of fraction, GOST R 50837.2-95 Peptization number, GOST R 50837.5-95	– –
Ductile lubricants	Index of penetration classes, GOST 5346-78	–
Hydrocarbons of the aromatic benzene series	Bromine number, GOST 2706.11-74	–
Paints	Acid number, GOST 23955-80	–
Lacquer, alkyd, and polyester resins	Hydroxyl number, GOST 26194-84	–
Polyesters for polyurethanes	Acid number, GOST 25210-82 Iodine value, GOST 25240-82 Hydroxyl number, GOST 26261-82	– – –
Anhydrous hardeners for epoxy resins	General acid number, GOST 25523-82	–
Methanol, technical poison	Permanganate number GOST 25742.5-83	–
Dark-colored raw material for PVA	Acid number, GOST 26028-83	–
Cellulose	Copper number, GOST 9418-75 Kapp number, GOST 10070-74	– –
Natural latex rubber	KOH number, GOST 28864-90	–
Rubber mixture ingredients, industrial carbon	Iodine value, GOST 25699.3-90	–
Bentonite clay for fine and building ceramics	Bentonite number, GOST 21282-93	–

Continuation of Table 3

Fruit and vegetable juices	Formol number, GOST R 51122-97	–
Essential oils and products of essential oil production	Acid number, GOST 30143-94 Ester number, GOST 30144-94	– –
Vegetable oils	Acid number, GOST 5476-80 Iodine value, GOST 5475-69 Peroxide number, GOST 26593-85	– – –
Sunflower	Acid number, GOST 26597-89	–
Higher fatty alcohols	Saponification and ester numbers, GOST 26549-85	–
Synthetic fatty acids	Ester number, GOST 22385-94	–
Cereal cultivation, grain and products of its processing	Settling number, GOST 30498-97, GOST 27676-88	–
Dimensions of figures of servicemen	Scale of sizes, GOST 20881-91	–
Woollen fabrics	Points of stability against moth damage, GOST 9.055-75	–
Wind force	Points on the Beaufort scale	–
Earthquake force	Points on the Richter scale	–
Accidents at nuclear power stations	Points on the IAEA scale	–

ative to its reference value (which possesses dimensionality). In the case of energy quantities, a bel, a decibel, and a neper signify increases by factors of respectively 10, 1.259, and 7.389. For force quantities, these factors are respectively 3.162, 1.121, and 2.719. The reference values for various quantities are usually selected from considerations of convenience, traditions, international agreements, etc. For example, in acoustics the reference values for the levels of sound intensity and acoustic pressure are chosen taking account of the psychophysiological properties of human hearing and are respectively 10^{-12} W/cm^2 and $2\cdot 10^{-5}$ Pa. In order to avoid errors, it is recommended by the standard MÉK 27-3-1974 that one should indicate the reference value following a specific numerical value of a level, for example an acoustic pressure level of 9 dB (relative to $2\cdot 10^{-5}$ Pa). It is thereby emphasized that this is a dimensional quantity which is conditionally presented as being dimensionless.

The pH value is a conditionally dimensionless quantity since by definition its value is obtained by dividing a dimensional quantity by a specific reference value of it and taking the logarithm of this dimensionless result of the division. Incidentally, we note that there is no requirement to introduce a special dimensionality or name for the pH unit since it makes no sense to attempt to use a pH value in conjunction with any property in order to form derived units.

Relative permittivity is also by definition a conditionally dimensionless quantity.

The example in Table 2 of the pitch of a musical sound is distinctive as a conditionally dimensionless quantity having a reference frequency of 440 Hz (the note A of the first octave). Octaves (the counter octave, first and second octaves, etc.) form a logarithmic scale and each of the octaves is marked out into a family of the well-known basic musical notes [19]. It should be mentioned that other forms of sound series exist for intervals in a musical octave. The scales of conditionally dimensionless quantities are naturally reproduced with reference to the SI system. In the State system for providing traceability of measurements there are State standards and devices of higher accuracy, originating in Russia, which reproduce the following conditional dimensionless quantities: the level of acoustic pressure in air and in water, pH and ionometric (pX) values, and relative permittivity.

Of particular importance is the case of numbers and scale points used to express measurements of quantities described by order scales (Table 3). It is well known [3–5, 8, 9] that on account of the undetermined nonlinearity in order scales (most frequently of the logical impossibility of establishing the proportionality of the quantities) it is meaningless to utilize the concept of a unit of the measured quantity. It is therefore also meaningless to accompany the expression of the results of measurements in such scales by words concerning units. The absence of units of measurement logically predetermines the lack of meaning in the question of the dimensionality of such quantities. Mathematical expressions used to calculate the values of numbers in terms of order scales are not definitive equations for dimensionality since they include not the interdependent properties of the measurement object but the parameters of a standardized measurement procedure (scale specifications): the parameters of experimental devices, those of factors acting on the measurement object, and those of the measurement action. The expression of the values of these parameters in dimensional units (such as SI units) is not justified in order to combine them into a meaningless unit of a quantity measured in terms of a scale. Thus, the procedure for measuring hardness using the Brinell scale is specified by the following parameters: the diameter (millimeters) of the indented hardened steel ball, the indentation force (newtons), the indentation duration (seconds), and the diameter (millimeters) of the impression of the ball on the surface of the metal sample.

In the light of what has been said about order scales, it is evidently totally meaningless to attempt to introduce a conditional unit for the acid number, for example, and arguments concerning the dimensionality of this "unit" [20, 21]. There is no justification for representing the procedure adopted to determine acid number values, in which KOH alkali is used to neutralize an acid group in a test sample of fat, in terms of a "determining equation" of the quantity and to consider the specific designation of this procedure (milligrams of KOH per gram) as a unit of measurement. If the scale of the acid number of fats is experimentally established to be linear, it will not even then be justified to introduce dimensionality for this number in view of the above discussion.

We share the concern of the authors of [22] over the need to legitimize measurements in terms of nonmetric scales, and in particular of measurements of an octave number (see Table 3). However, we do not share the proposal of introducing "conditional units" with a normative document. The explanations presented above demonstrate the error of this proposal. In this case, there is no justification or need to introduce "conditional units."

Unfortunately, the mistaken proposals considered, and other such proposals, related to numbers and scale points of an order scale are far from unique. A rather preliminary choice of active order scales is given specially in Table 3 in order to raise awareness of the scale and seriousness of the problem. Many of these scales are regulated not only by State standards but also by international standards of, for example, scales of hardness, light sensitivity, settling number, classifications of accidents at nuclear power stations, etc. However, some socially significant order scales in Russia are not standardized at the State level (scales of wind force, earthquake force, nuclear power station accident classification).

Table 3 brings to light multiple repetition of various hardness numbers, acid numbers, iodine values, etc. Formally operating standardizers have a desire to eliminate such repetition and to introduce (for example in [20, 21]) one general acid number for different objects. This desire reveals a misunderstanding of the fundamental impossibility of such a unification. The acid numbers of different objects, even in the case when they are measured using similar procedures, are the results using different scales of measurements which cannot be compared. For example, the formal equality of the acid number values of petroleum products and vegetable oils does not equalize the useful properties of these objects. There is also no sense in comparing with each other the hardness of steel, rubber, and plastics. Even for a fairly homogeneous class of objects, metals and alloys, several hardness scales exist each of which describes the hardness of the material in accordance with its specific definition. It is shown in practice that all these have their own range of application.

Unfortunately, there are inaccuracies in several of the standards shown in Table 3 which it would be desirable to eliminate when reviewing them. For example, in certain standards for hardness scales comment is made of conditional or relative units, or the results of measurements are accompanied by the notation MPa, N/m^2, or kg/mm^2 which are understood as units of measurement. Frequently it is not the median of the measurements which is used, as should be the case in order scales, but the arithmetic mean of the observations. The recommendations of [9] will be useful in eliminating these and other shortcomings in the normative documents on order scales.

The majority of the order scales given in Table 3 have been subjected to metrological assurance. A number of State standards of hardness scales exist with corresponding State verification schemes. When necessary, measuring instruments are tested for the purpose of confirming their type and they are entered into the State register of measuring instruments. Thus, in recent years tests have been made of several models of an instrument for measuring the settling number, one of the characteristics of flour, and they have been entered in the State register. For many scales, the measuring procedures are carried out using legitimized measuring instruments borrowed from other forms of measurement. Some order scales are of no interest for instrumental metrology, although they are necessary in order to perform State metrological inspection. Examples of this are the dimensions of the figures of servicemen or the stability scale for woollen fabrics against moth damage, etc.

The above discussion provides evidence that dimensionless units and numbers represent an extremely diverse set which is successfully utilized for presenting the results of measurements. There is no need to attempt to introduce them into any "system of units." The realization of this fact leads to a more satisfactory understanding of the real structure of metrology.

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